Maturitetne naloge

z rešitvami

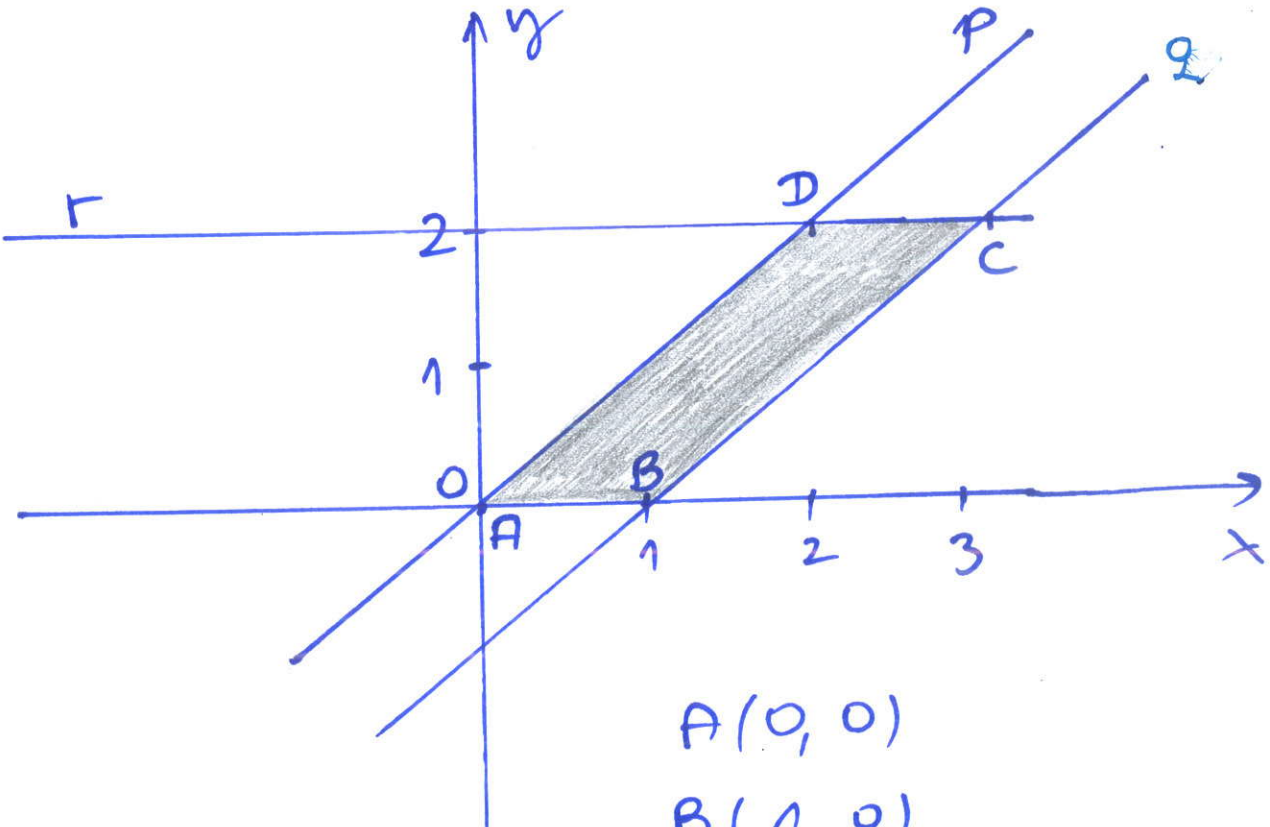
3. junij 2017

Osnovna raven ki je

hkrati prva pola

višje ravni.

1. V pravokotnem koordinatnem sistemu v ravnini so narisane premice p, q in r. Te tri premice z abscisno osjo oklepajo paralelogram ABCD. Zapišite enačbe premic ter izračunajte ploščino in obseg paralelograma. Rezultata naj bosta točna. (7 točk)



A(0, 0)

B(1, 0)

C(3, 2)

D(2, 2)

Rešitev: $k = \frac{y_2 - y_1}{x_2 - x_1}$ $y - y_1 = k(x - x_1)$

Premica p:

$A(0,0)$ $k = \frac{2-0}{2-0} = \frac{2}{2} = 1$

$D(2,2)$ $y - 0 = 1(x - 0)$

$\boxed{y = x}$ (1 točka)

Premica q:

$B(1,0)$ $k = \frac{2-0}{3-1} = \frac{2}{2} = 1$

$C(3,2)$ $y - 0 = 1(x - 1)$

$\boxed{y = x - 1}$ (1 točka)

Vodoravna premica r:

$\boxed{y = 2}$ (1 točka)

Osnovnica paralelograma ABCD je

$$a = d(A,B) = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$
$$= \sqrt{(1-0)^2 + (0-0)^2} = \sqrt{1+0} = 1$$

višina paralelograma ABCD: $h=2$

$$S_{ABCD} = a \cdot h = 1 \cdot 2$$

$$\boxed{S_{ABCD} = 2}$$ (2 točki)

$$d(B,C) = \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2} =$$

$$= \sqrt{(3-1)^2 + (2-0)^2} =$$

$$= \sqrt{2^2 + 2^2} = \sqrt{2 \cdot 4} = 2\sqrt{2}$$

$$d(C,D) = 1$$

$$d(A,D) = d(B,C) = 2\sqrt{2}$$

obseg paralelograma =

$$= |AB| + |BC| + |CD| + |AD| =$$

$$= 1 + 2\sqrt{2} + 1 + 2\sqrt{2} =$$

$$= 2 + 4\sqrt{2}$$

$$\boxed{o = 2 + 4\sqrt{2}}$$ (2 točki)

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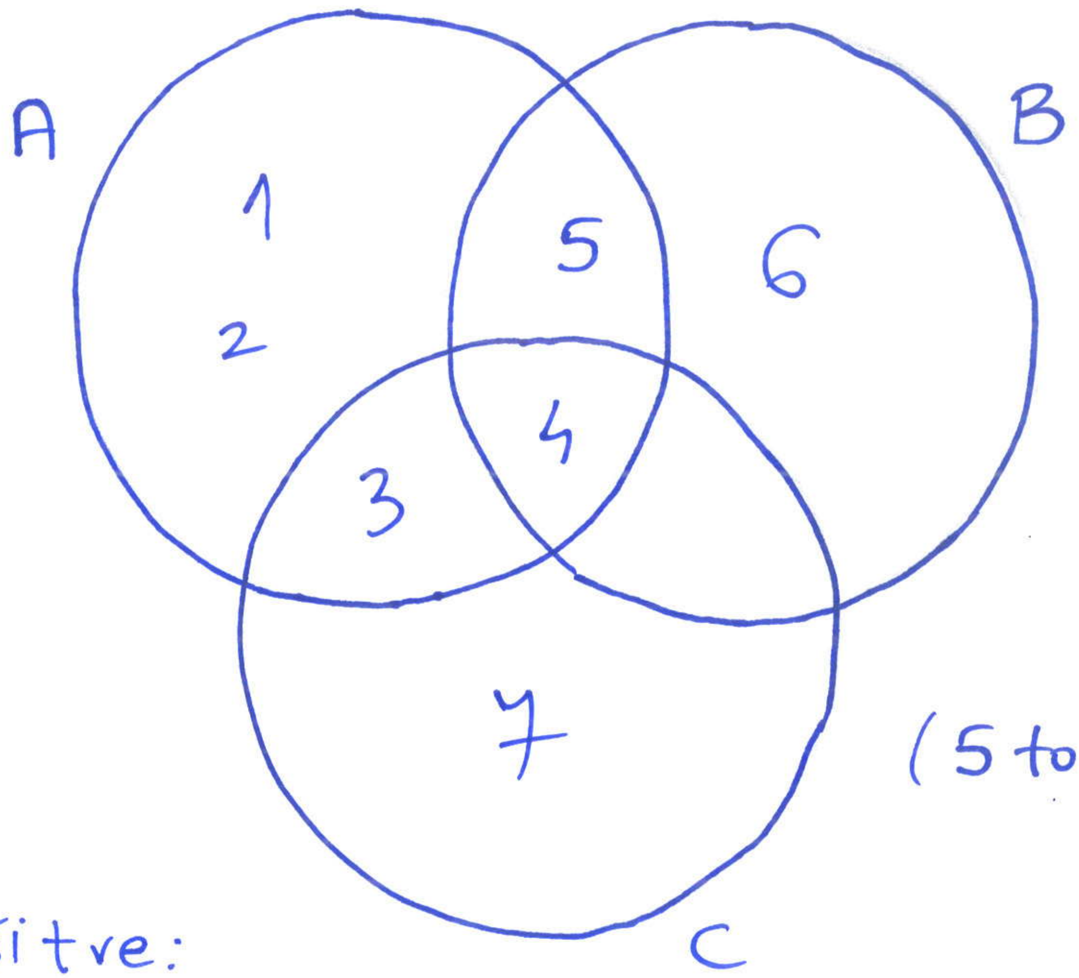
2. Na sliki so narisane množice A, B in C. Zapišite množice z naštevanjem elementov.



(5 točk)

Rešitve:

$A = \{1, 2, 3, 4, 5\}$

$B \cap C = \{4\}$

$B \cup A = \{1, 2, 3, 4, 5, 6\}$

$A - C = \{1, 2, 5\}$

$B \times (A \cap B \cap C) = \{4, 5, 6\} \times \{4\} =$

$= \{(4, 4), (5, 4), (6, 4)\}$

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3. Rešite enačbe. Rezultati naj bodo točni:

3.1. $x^2+2x=4$ (2)

3.2. $4^x=2$ (1)

3.3. $\log_4 x=2$ (1)

3.4. $4\sin x=2$ (3)

(7 točk)

Rešitev:

3.1. $x^2+2x-4=0$

$a=1 \quad b=2 \quad c=-4$

$D=b^2-4ac=(2)^2-4\cdot 1\cdot(-4)=$

$=4+16=20$

$\sqrt{D}=\sqrt{20}=2\sqrt{5}$

$x_{1/2}=\frac{-b\pm\sqrt{D}}{2a}=\frac{-2\pm 2\sqrt{5}}{2}$

$$\boxed{\begin{aligned} x_1 &= -1-\sqrt{5} \\ x_2 &= -1+\sqrt{5} \end{aligned}}$$

3.2. $4^x=2$

$(2^2)^x=2^1$

$2x=1$

$\boxed{x=\frac{1}{2}}$

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3.3. $\log_4 x = 2$

$x = 4^2$

$\boxed{x = 16}$

3.4. $4\sin x = 2 \quad /:4$

$\sin x = \frac{2}{4}$

$\sin x = \frac{1}{2}$

$x_1 = \arcsin\left(\frac{1}{2}\right) + k \cdot 2\pi$

$\boxed{x_1 = \frac{\pi}{6} + k \cdot 2\pi}$ $\boxed{k \in \mathbb{Z}}$

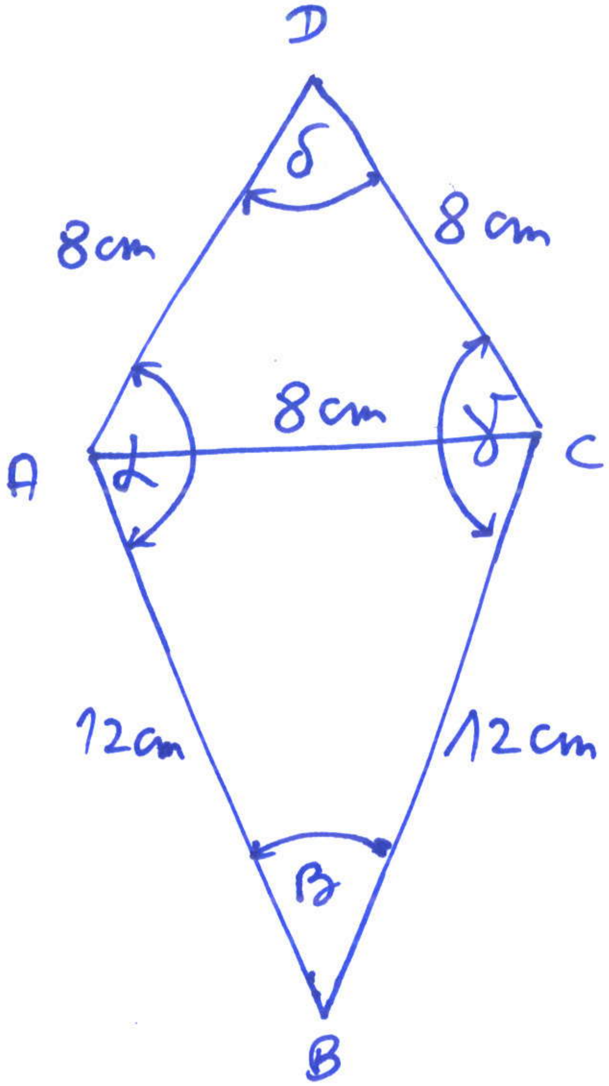
$x_2 = \pi - \arcsin\left(\frac{1}{2}\right) + k \cdot 2\pi$

$x_2 = \pi - \frac{\pi}{6} + k \cdot 2\pi$

$\boxed{x_2 = \frac{5\pi}{6} + k \cdot 2\pi}$

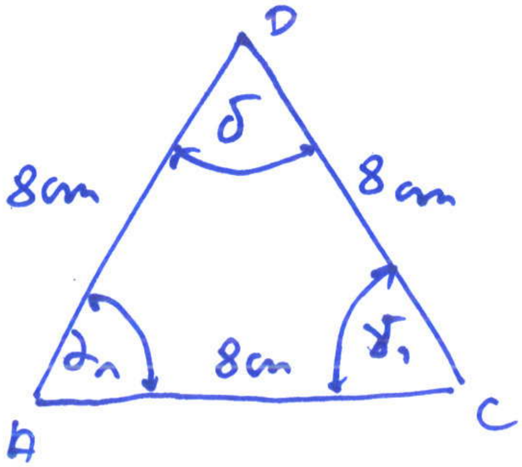
4. Izračunajte velikosti notranjih kotov štirikotnika ABCD in dolžino diagonale $f = |BD|$.

(8 točk)



Rešitev:

Trikotnik ACD je enakostraničen.

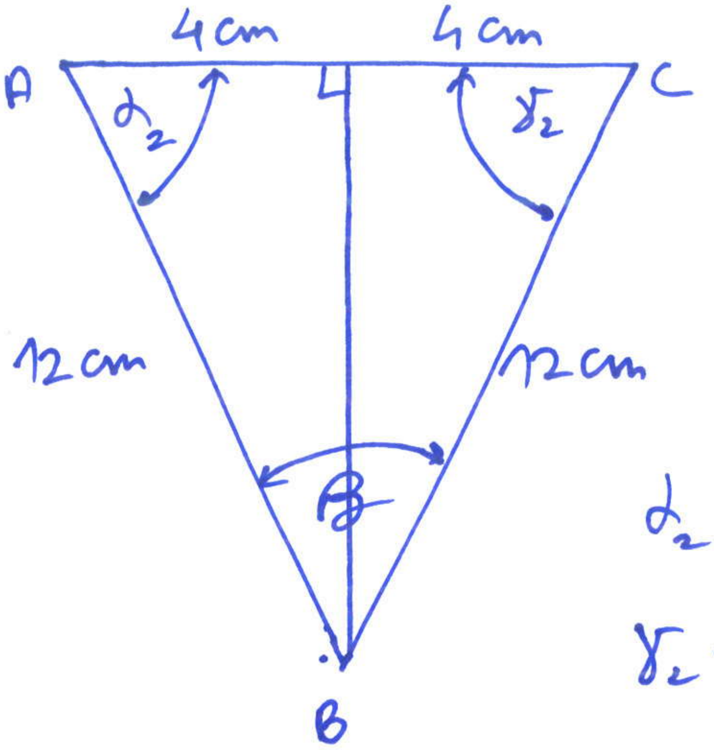


$$\boxed{\delta = 60^\circ}$$

$\alpha_1 = 60^\circ$

$\gamma_1 = 60^\circ$

Trikotnik ABC je enakokrak.



$$\cos\alpha_2 = \frac{4}{12}$$

$$\alpha_2 = \arccos\left(\frac{1}{3}\right)$$

$$\alpha_2 = 70{,}53^\circ = 70^\circ 32'$$

$$\gamma_2 = \alpha_2$$

$$\alpha = \alpha_1 + \alpha_2 = 60^\circ + 70{,}53^\circ$$

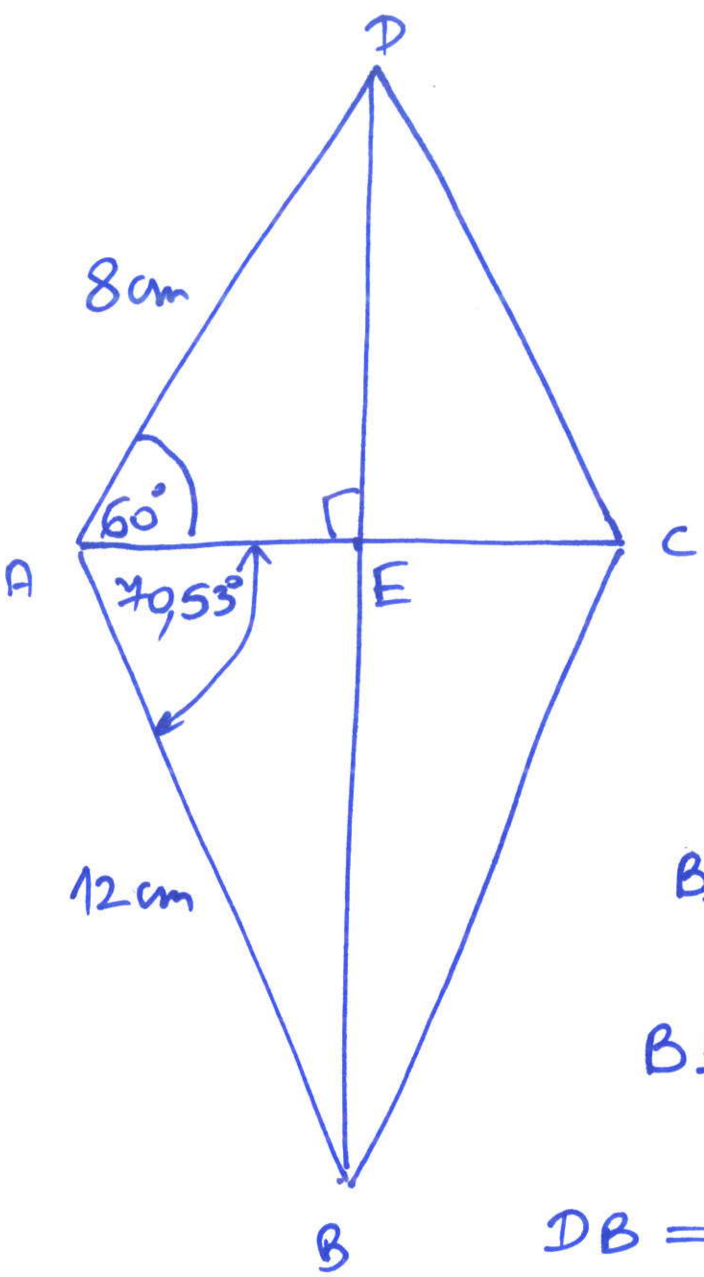
$$\boxed{\alpha = 130{,}53^\circ = 130^\circ 32'}$$

$$\boxed{\gamma = \alpha = 130{,}53^\circ = 130^\circ 32'}$$

$$\beta = 180 - \alpha_2 - \gamma_2 =$$

$$= 180 - 70{,}53^\circ - 70{,}53^\circ$$

$$\boxed{\beta = 38{,}94^\circ = 38^\circ 57'}$$



$$\frac{DE}{8} = \sin 60°$$

$$DE = 8 \cdot \sin 60$$

$$DE = 6{,}93\ cm$$

$$\frac{BE}{12} = \sin 70{,}53°$$

$$BE = 12 \cdot \sin 70{,}53°$$

$$BE = 11{,}31\ cm$$

$$DB = DE + EB$$

$$DB = 6{,}93 + 11{,}31$$

$$DB = 18{,}24\ cm$$

$$\boxed{f = 18{,}24\ cm}$$

5. Naj bo $z = x(4-3i) + 5i + i^2$.

$z \in \mathbb{C}$. Izračunajte realno število x

tako, da bo veljalo $Re(z) = Im(z)$.

(5 točk)

Rešitev:

$$z = x(4-3i) + 5i + i^2$$

$$z = 4x - 3xi + 5i - 1$$

$$z = +4x - 1 + 5i - 3xi$$

$$z = (4x-1) + i(5-3x)$$

$$Re(z) = Im(z)$$

$$(4x-1) = (5-3x)$$

$$4x + 3x = 5 + 1$$

$$7x = 6$$

$$\boxed{x = \frac{6}{7}}$$

6. V prostoru $\mathbb{R}^3$ so dani vektorji $\vec{a} = (1, 2, -1)$, $\vec{b} = (3, -2, -1)$ in $\vec{c} = (1, 1, 2)$.

6.1. Računsko pokažite, da sta vektorja $\vec{a}$ in $\vec{b}$ pravokotna. (2)

6.2. Izračunajte dolžini vektorjev $\vec{a}$ in $\vec{c}$ ter velikost kota φ med njima. Velikost kota zaokrožite na dve decimalni mesti (5)

(7 točk)

Rešitev:

6.1. $\vec{a} \cdot \vec{b} = (1, 2, -1) \cdot (3, -2, -1) =$

$= 1 \cdot 3 + 2(-2) + (-1)(-1) =$

$= 3 - 4 + 1 = 0$

$\boxed{\vec{a} \cdot \vec{b} = 0}$ Vektorja sta pravokotna.

6.2. $|\bar{a}| = \sqrt{\vec{a} \cdot \bar{a}} = \sqrt{(1,2,-1)(1,2,-1)} =$

$= \sqrt{1 \cdot 1 + 2 \cdot 2 + (-1) \cdot (-1)} = \sqrt{1+4+1} = \sqrt{6}$

$$\boxed{|\bar{a}| = \sqrt{6}}$$

$|\bar{c}| = \sqrt{\vec{c} \cdot \bar{c}} = \sqrt{(1,1,2) \cdot (1,1,2)} =$

$= \sqrt{1 \cdot 1 + 1 \cdot 1 + 2 \cdot 2} = \sqrt{1+1+4} = \sqrt{6}$

$$\boxed{|\bar{c}| = \sqrt{6}}$$

$\vec{a} \cdot \bar{c} = (1,2,-1) \cdot (1,1,2) =$

$= 1 \cdot 1 + 2 \cdot 1 + (-1) \cdot 2 = 1 + 2 - 2 = 1$

$$\cos\varphi = \frac{\bar{a} \cdot \vec{c}}{|\bar{a}| \cdot |\bar{c}|} = \frac{1}{\sqrt{6} \cdot \sqrt{6}}$$

$$\cos\varphi = \frac{1}{6}$$

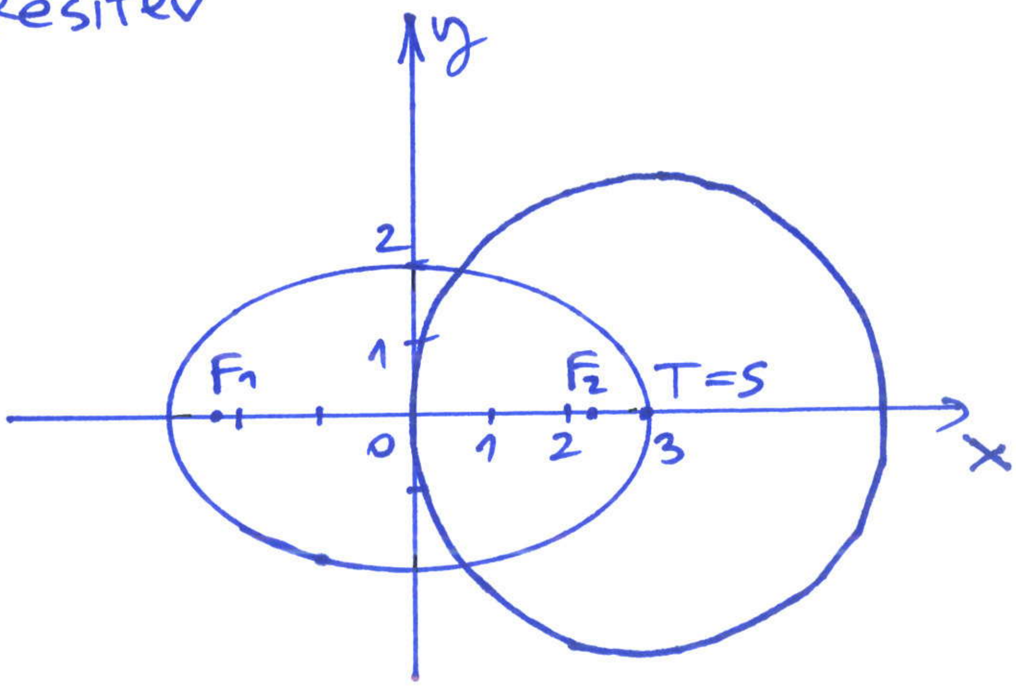
$$\varphi = \arccos\left(\frac{1}{6}\right)$$

$$\boxed{\varphi = 80{,}41^\circ}$$

7. V doni koordinatni sistem narišite elipso z enačbo $4x^2+9y^2-36=0$. Zapišite gorišča elipse. Zapišite enačbo krožnice, ki ima središče v desnem temenu dane elipse in se dotika ordinatne osi.

(7 točk)

Rešitev



$$4x^2+9y^2=36 \;/:36$$

$$\frac{4x^2}{36}+\frac{9y^2}{36}=1 \qquad \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{x^2}{9}+\frac{y^2}{4}=1$$

$a^2 = 9$ $\qquad$ $b^2 = 4$

$a = 3$ $\qquad$ $b = 2$

$$e = \sqrt{a^2 - b^2} = \sqrt{9-4} = \sqrt{5} \cong 2{,}24$$

$F_1(-e, 0)$ $\qquad$ $F_2(e, 0)$

$\boxed{F_1(-\sqrt{5}, 0)}$ $\qquad$ $\boxed{F_2(\sqrt{5}, 0)}$

Desno teme elipse $T(3, 0)$

je središće kroŽnice $S(3, 0)$

Polmer kroŽnice $r = 3$

$$(x-p)^2 + (y-q)^2 = r^2$$

$$(x-3)^2 + (y-0)^2 = 3^2$$

$$\boxed{(x-3)^2 + y^2 = 9}$$

8. Izračunajte za kateri x so x^2-3, $x-1$ in $1-2x$ zaporedni členi aritmetičnega zaporedja.

(5 točk)

Rešitev:

$$a_2 - a_1 = a_3 - a_2$$

$$(x-1)-(x^2-3) = (1-2x)-(x-1)$$

$$x-1-x^2+3 = 1-2x-x+1$$

$$-x^2+x+2 = -3x+2$$

$$-x^2+x+2+3x-2=0$$

$$-x^2+4x=0 \quad /\cdot(-1)$$

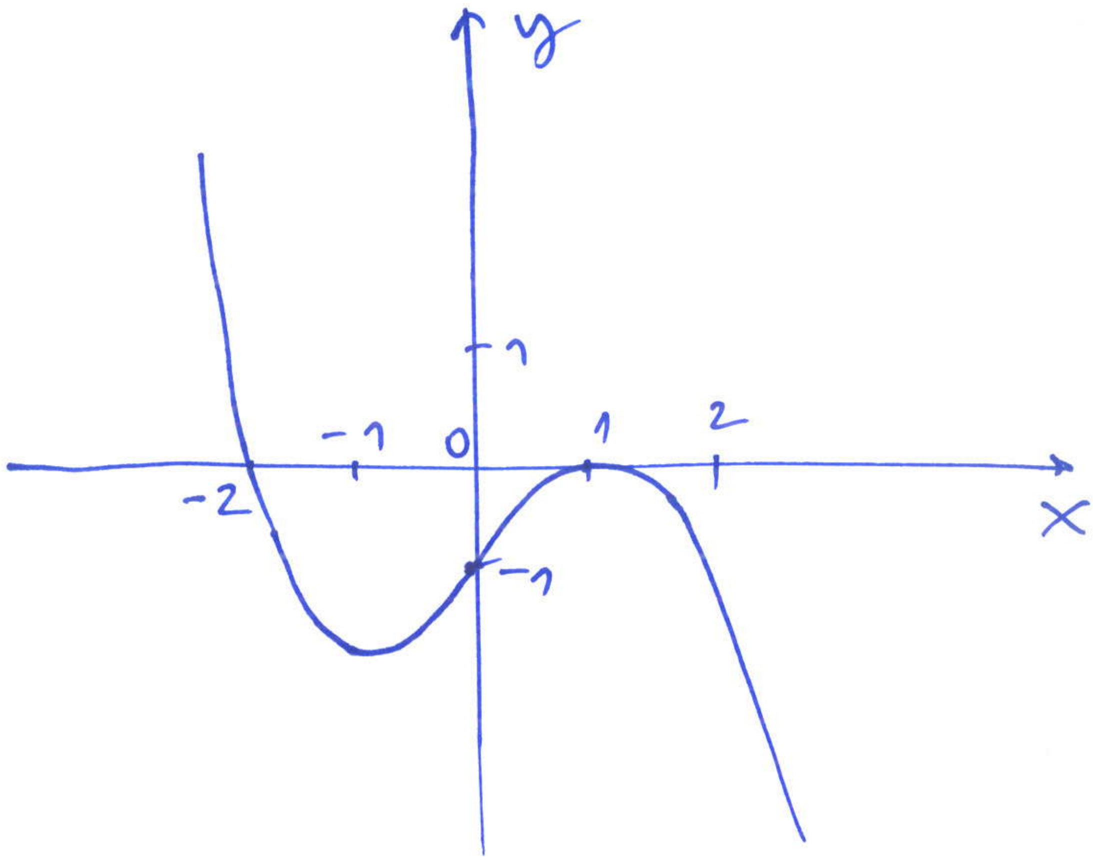
$$x^2-4x=0$$

$$x(x-4)=0$$

$$\boxed{x_1=0} \qquad \boxed{x_2=4}$$

A. Z.: $-3, -1, 1$ A. Z. $13, 3, -7$

9. Na sliki je graf polinoma $p(x)$ tretje stopnje.



9.1. Zapišite polinom v ničelni obliki (5)

9.2. V koordinatni sistem narišite graf polinoma

$$f(x) = p(x) + 1 \qquad (1)$$

(6 točk)

Rešitev:

Ničle: $x_1 = -2$ prva stopnja

$x_2 = 1$ druga stopnja

$$y = a(x - x_1)^1 \cdot (x - x_2)^2$$

$$y = a(x+2) \cdot (x-1)^2$$

Graf poteka skozi točko $A(0,-1)$

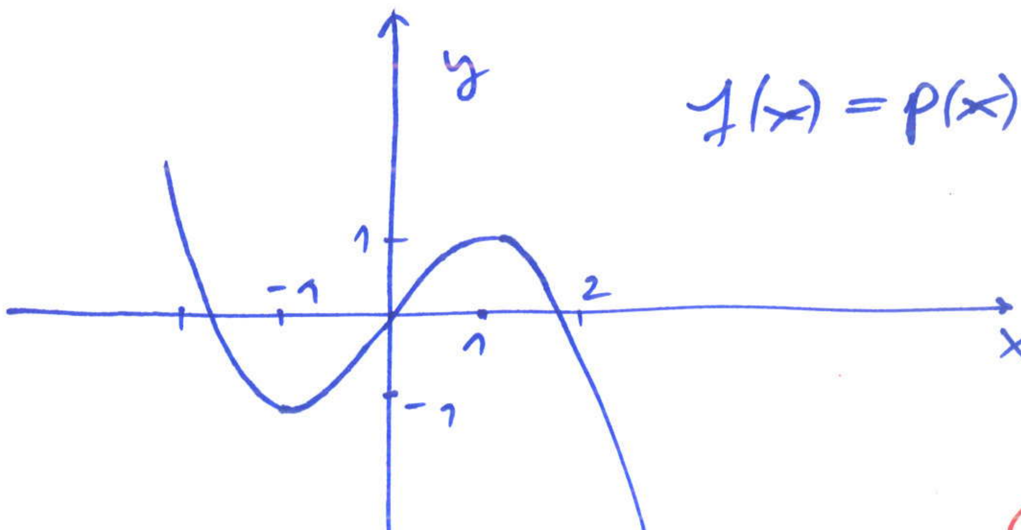
$$-1 = a(0+2)(0-1)^2$$

$$-1 = a \cdot 2 \cdot 1 \quad /:2$$

$$-\frac{1}{2} = a$$

$$\boxed{y = -\frac{1}{2}(x+2) \cdot (x-1)^2}$$

9.2



$f(x) = p(x) + 1$

10. Racionalna funkcija ima predpis $f(x) = \frac{x^2+3}{x+1}$. Zapišite stacionarni točki $S_1(x_1, y_1)$ in $S_2(x_2, y_2)$. V kateri točki ima funkcija lokalni minimum in maksimum? Odgovor utemeljite.

Rešitev:

$$y' = \left(\frac{x^2+3}{x+1}\right)' = \frac{(x^2+3)'\cdot(x+1) - (x^2+3)(x+1)'}{(x+1)^2} =$$

$$= \frac{(2x)(x+1) - (x^2+3)(1)}{(x+1)^2} =$$

$$= \frac{2x^2+2x - x^2-3}{(x+1)^2} =$$

$$y' = \frac{x^2+2x-3}{(x+1)^2}$$

$$y' = 0 \qquad x^2+2x-3 = 0$$

$$(x+3)(x-1) = 0$$

$x+3=0 \qquad\qquad x-1=0$

$x_1=-3 \qquad\qquad x_2=1$

$$f(-3)=\frac{(-3)^2+3}{(-3)+1}=\frac{9+3}{-3+1}=\frac{12}{-2}$$

$f(-3)=-6 \qquad\qquad S_1(-3,-6)$

$$f(1)=\frac{1^2+3}{1+1}=\frac{4}{2}=2$$

$S_2(1,2)$

Predznak prvega odvoda je

	-3		1		
$+\ +$	|	$-\ -$	|	$+\ +$	y'

V točki $S_1(-3,-6)$ je lokalni maksimum

V točki $S_2(1,2)$ je lokalni minimum.

11. Dani sta realni funkciji $f(x)$ in $g(x)$ s predpisoma

$$f(x) = x^2 \quad \text{in} \quad g(x) = 6 - x.$$

Izračunajte ploščino lika ki ga omejujeta grafa funkcij $f(x)$ in $g(x)$.

(7 točk)

Rešitev:

$$x^2 = 6 - x$$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

$$x + 3 = 0 \qquad x - 2 = 0$$

$$x_1 = -3 \qquad x_2 = 2$$

$$S = \int_{x_1}^{x_2} [g(x) - f(x)]\, dx$$

$$S = \int_{-3}^{2} \left[(6-x) - (x^2) \right] dx =$$

$$= \int_{-3}^{2} (6 - x - x^2)\, dx =$$

$$= \left(6x - \frac{x^2}{2} - \frac{x^3}{3} \right) \Bigg|_{-3}^{2} =$$

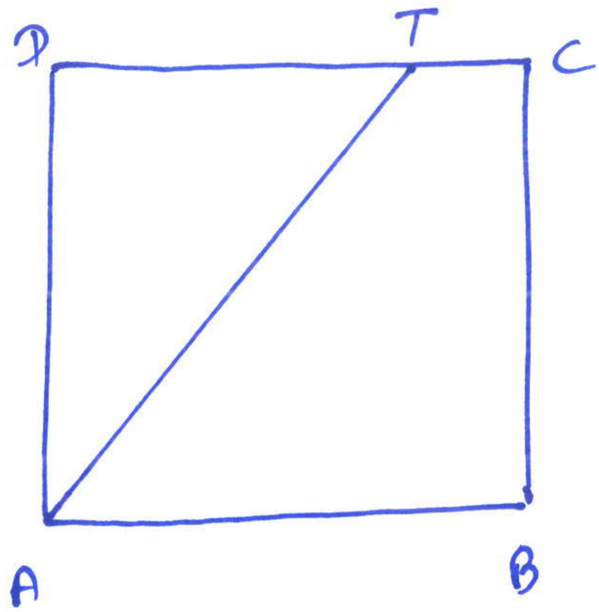
$$= \left(6 \cdot 2 - \frac{2^2}{2} - \frac{2^3}{3} \right) - \left(6(-3) - \frac{(-3)^2}{2} - \frac{(-3)^3}{3} \right) =$$

$$= \left(12 - 2 - \frac{8}{3} \right) - \left(-18 - \frac{9}{2} + 9 \right) =$$

$$= 12 - 2 - \frac{8}{3} + 18 + \frac{9}{2} - 9$$

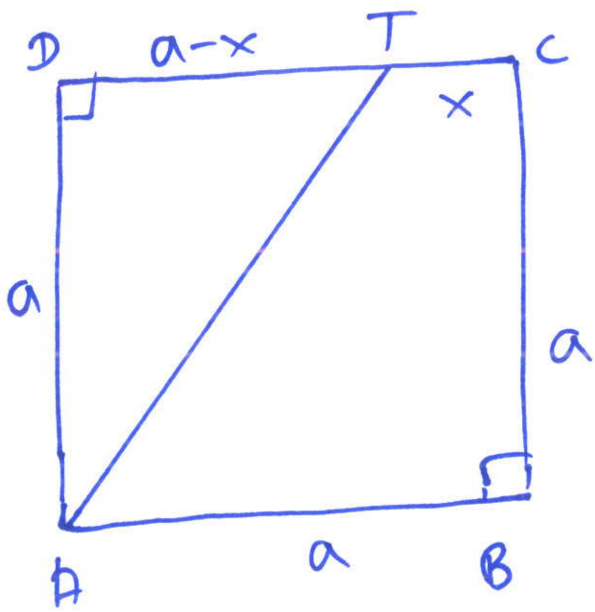
$$\boxed{S = \frac{125}{6} = 20{,}83}$$

12. V kvadratu s stranico a je narisana daljica AT tako da je razmerje ploščin likov $2:3$. Izračunajte razmerje dolžin $|DT|:|TC|$.



$$S_{ATD} : S_{ABCT} = 2:3$$

Rešitev:



$TC = x$

$DT = a - x$

Trikotnik ATD ima ploščino

$$S_{ATD} = \frac{1}{2} a\cdot(a-x)$$

Trapez $ABCT$ ima ploščino $S_{ABCT} = \frac{1}{2}(a+x)\cdot a$

$$\frac{S_{ATD}}{S_{ABCT}} = \frac{2}{3}$$

$$\frac{\frac{1}{2}a(a-x)}{\frac{1}{2}(a+x)\cdot a} = \frac{2}{3}$$

$$\frac{(a-x)}{(a+x)} = \frac{2}{3} \quad / \cdot 3(a+x)$$

$$3(a-x) = 2(a+x)$$

$$3a - 3x = 2a + 2x$$

$$3a - 2a = 3x + 2x$$

$$a = 5x$$

$$x = \frac{a}{5} \qquad DT = a - x =$$

$$= a - \frac{a}{5} = \frac{4}{5}a$$

$$\frac{DT}{TC} = \frac{\frac{4}{5}a}{\frac{a}{5}} = \frac{4}{1}$$

$$\boxed{|DT| : |TC| = 4 : 1}$$

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